

# New constraints on the Yukawa-type corrections to Newtonian gravity at short separations

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## Abstract

We discuss the strongest constraints on the Yukawa-type corrections to Newton's gravitational law within a submicrometer interaction range following from measurements of the Casimir force. In this connection the complicated problems arising when comparing the measurement data with the Lifshitz theory are analyzed. Special attention is paid to the results of two recent experiments on measuring the Casimir interaction between ferromagnetic surfaces and sinusoidally corrugated surfaces at various angles between corrugations.

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## I. INTRODUCTION

Among all fundamental interactions gravitation is the most commonly known and simultaneously the most difficult for both experimental and theoretical investigation. It might be considered paradoxical that up to the present the gravitational constant is measured with less precision than other fundamental physical constants. During the last century research in gravitation was somewhat isolated from all other branches of physics dealing mostly with quantum phenomena. Many attempts were undertaken to combine gravitation with other interactions in the framework of some unified description, e.g., supergravity, but all of them till the moment are only impressive mathematical schemes rather than successful physical theories. Against this background a few achievements presenting consistent and physically reasonable unification between gravitation and quantum phenomena in some special cases are of even greater value.

One of these achievements is the quantum field theory in spatially homogeneous isotropic space-time developed by Prof. A. A. Grib and his collaborators (see papers [1–6], review [7] and monograph [8]). This theory was applied to the Friedmann cosmological models describing our Universe and found a lot of prospective applications to the effects of particle creation from vacuum by the nonstationary gravitational field, polarization of vacuum and spontaneous symmetry breaking. On similar grounds Prof. A. A. Grib and his collaborators developed the theory of particle creation by a nonstationary electric field [9, 10] (recently the same methods were applied [11] to describe the creation of quasiparticles in graphene).

Taking into account the lack of experimental information on the border between gravitational physics and quantum phenomena, much attention has been recently paid to the search of Yukawa-type corrections to Newtonian gravity at short separations [12]. Such corrections arise due to exchange of hypothetical light elementary particles predicted [13] by many extensions of the Standard Model and in extra-dimensional physics with low-energy compactification scale [14]. In the range of separations between the test bodies below a few micrometers Newton's law of gravitation is not verified experimentally, so that corrections to it are possible which exceed gravity by many orders of magnitude. These corrections cannot be constrained with the help of standard gravitational experiments of Eötvös and Cavendish type. The point is that at so small separations the van der Waals and Casimir forces which act between the closely spaced surfaces of probe masses due to electromagnetic

fluctuations become much larger than the gravitational force. The latter loses its sensitivity to the presence of possible corrections. Because of this in the interaction range below a few micrometers it was proposed [15, 16] to use measurements of the van der Waals and Casimir force for obtaining stronger constraints on the corrections to Newton's gravitational law.

During the last 15 years a lot of experiments on measuring the Casimir force between metallic, semiconductor and dielectric test bodies has been performed [17, 18]. As a result, the previously known constraints on the Yukawa-type corrections to Newtonian gravity in the submicrometer interaction range were strengthened up to a factor of 24 millions [19]. In doing so unexpected theoretical problems related to the comparison between experiment and theory have been analyzed. In the present paper we briefly summarize the strongest constraints on the Yukawa-type corrections to Newtonian gravity following from measurements of the Casimir force. We discuss the reliability of these constraints in connection with abovementioned problems arising in the fluctuational electrodynamics. We also present the most recent constraints obtained from two experiments performed in 2013.

The paper is organized as follows. In Sec. II we introduce the used notations and parametrizations and summarize the results obtained in the past. Section III is devoted to the comparison between measurements of the Casimir force and theory. In Sec. IV we present the constraints on corrections to Newton's law obtained from two most recent experiments on measuring the Casimir force between ferromagnetic and corrugated surfaces. Section V is devoted to our conclusions and discussion.

## II. YUKAWA-TYPE CORRECTIONS TO NEWTON'S GRAVITATIONAL LAW AND CONSTRAINTS ON THEM FROM MEASUREMENTS OF THE CASIMIR FORCE

It is conventional to present the gravitational potential between the two pointlike masses  $m_1$  and  $m_2$  spaced at a separation  $r$  as a sum of the Newtonian part  $V_N(r)$  and the Yukawa-type correction  $V_{\text{Yu}}(r)$ :

$$V(r) = V_N(r) + V_{\text{Yu}}(r) = -\frac{Gm_1m_2}{r} (1 + \alpha e^{-r/\lambda}). \quad (1)$$

Here,  $G$  is the Newtonian gravitational constant, and  $\alpha$  and  $\lambda$  are the strength and interaction range of the Yukawa-type correction. If the Yukawa-type correction to Newtonian

gravitational potential  $V_N(r)$  is caused by an exchange of light bosons of mass  $m$  between the probe masses  $m_1$  and  $m_2$ , the interaction range  $\lambda$  has the meaning of the Compton wavelength of this boson  $\lambda = \hbar/(mc)$ . Alternatively, if the Yukawa-type correction arises due to the compactification of extra spatial dimensions in multi dimensional schemes, the quantity  $\lambda$  has the physical meaning of the size of the compact manifold.

As was mentioned in Sec. I, at separations below a few micrometers the Newtonian gravitational force becomes smaller than the van der Waals and Casimir forces acting between closely spaced surfaces. In fact it is even smaller than the error in the measurements of the van der Waals forces. Because of this, when calculating the interaction energy of two macroscopic bodies due to potential (1), one can neglect by the Newtonian contribution and integrate the Yukawa-type correction alone over the volumes of both bodies

$$V_{\text{Yu}}(a) = -G\alpha \int_{V_1} d^3r_1 \rho_1(\mathbf{r}_1) \int_{V_2} d^3r_2 \rho_2(\mathbf{r}_2) \frac{e^{-|\mathbf{r}_1 - \mathbf{r}_2|/\lambda}}{|\mathbf{r}_1 - \mathbf{r}_2|}. \quad (2)$$

Here,  $\rho_1(\mathbf{r}_1)$  and  $\rho_2(\mathbf{r}_2)$  are the mass densities of, generally speaking, nonhomogeneous test bodies and  $a$  is the closest separation between them. Then the Yukawa-type force and its gradient are given by

$$F_{\text{Yu}}(a) = -\frac{\partial V_{\text{Yu}}(a)}{\partial a}, \quad \frac{\partial F_{\text{Yu}}(a)}{\partial a} = -\frac{\partial^2 V_{\text{Yu}}(a)}{\partial a^2}. \quad (3)$$

In the experiments on measuring the Casimir force (see reviews in Refs. [17, 18]) either the force  $F_C(a, T)$  acting between two test bodies or its gradient  $\partial F_C(a, T)/\partial a$  have been measured ( $T$  is the temperature at which measurements under consideration are performed). The measurement results were compared with theoretical predictions for the Casimir force and its gradient and good agreement was found in the limits of some experimental errors  $\Delta_{F_C}(a)$  and  $\Delta_{F'_C}(a)$ , respectively. Thus, within the limits of these errors no hypothetical Yukawa-type interaction was observed. The respective constraints on the parameters of Yukawa interaction  $\alpha$ ,  $\lambda$  follow from the inequalities

$$|F_{\text{Yu}}(a)| \leq \Delta_{F_C}(a), \quad \left| \frac{\partial F_{\text{Yu}}(a)}{\partial a} \right| \leq \Delta_{F'_C}(a), \quad (4)$$

where  $\alpha$ - and  $\lambda$ -dependent expression for  $F_{\text{Yu}}(a)$  is given by Eqs. (2) and (3).

Now we list the most strong constraints on the parameters  $\alpha$  and  $\lambda$  obtained from measurements of the Casimir force performed before 2013. The constraints on  $\alpha$  and  $\lambda$  are usually presented as some lines in the  $(\lambda, \alpha)$  plane where the region of this plane above

the line is prohibited by the results of respective experiment and the region below the line is allowed. By line 1 in Fig. 1 we show constraints obtained [20, 21] from measurements of the Casimir force between Au-coated surfaces of a microsphere and a plate by means of an atomic force microscope [22]. Line 2 in the same figure shows constraints obtained from measuring the gradient of the Casimir force between similar surfaces by means of a micromachined oscillator [20, 23]. Line 3 in Fig. 1 follows from the so-called Casimir-less experiment where the contribution of the Casimir force acting between a microsphere and a plate was compensated using some special arrangement of the setup [24]. Finally, line 4 shows the constraints obtained [25] from measurements of the Casimir force between Au-coated surfaces of a plate and a spherical lens of centimeter-size radius of curvature by means of torsion pendulum.

As can be seen in Fig. 1, the strength of constraints following from the Casimir effect quickly increases with the increase of  $\lambda$ . However, for  $\lambda$  exceeding several micrometers the strongest constraints on  $\alpha$  and  $\lambda$  follow not from measurements of the Casimir force but from gravitational experiments. To illustrate this, in Fig. 1 we plot by the line 5 the constraints obtained from the most precise Cavendish-type experiment of Refs. [26, 27]. At the same time, with decreasing  $\lambda$  down to 1 nm the strength of constraints shown by the lines 1–4 quickly decreases. It was shown [19] that within the interaction range from 1.6 to 14 nm the strongest constraints on  $\alpha$ ,  $\lambda$  follow from measurements of the lateral Casimir force which arises between sinusoidally corrugated surfaces of a sphere and a plate with common period [28, 29]. These constraints are shown by the line 6 in Fig. 1. At even shorter  $\lambda$  below 1 nm the strongest constraints on the Yukawa-type corrections to Newtonian gravity follow from precision atomic physics [30].

In the end of this section it is worth noting that in Fig. 1 we do not show constraints obtained from measurements of the Casimir force with the help of torsion pendulum presented in Refs. [31–34]. The point is that these are not direct measurements of the Casimir force, but of much larger force of unknown nature from which the Casimir contribution was extracted by means of the fitting procedure using some postulated theoretical expressions. The critical discussion of these experiments contained in the literature [17, 18, 35–39] leads to a conclusion that both the measured data and respective constraints are not reliable.

### III. PROBLEMS IN EXPERIMENT-THEORY COMPARISON FOR THE CASIMIR FORCE

As explained in Sec. II, the strongest constraints on the Yukawa-type corrections to Newtonian gravity in the submicrometer interaction range are obtained from the measure of agreement between the experimental data and theoretically calculated Casimir forces. Because of this, both the solid data and consistent theory are required for the reliability of these constraints. The fundamental theory of the van der Waals and Casimir forces used in calculations was developed by Lifshitz [40] in the framework of fluctuational electrodynamics and is commonly known as the Lifshitz theory. In this theory the Casimir free energy  $\mathcal{F}_C(a, T)$  and force  $F_C(a, T)$  are expressed via the frequency-dependent dielectric permittivities of the interacting bodies. For Au bodies used in most of experiments the complex index of refraction  $n(\omega)$  [and respective dielectric permittivity  $\varepsilon(\omega)$ ] is measured over a wide range of frequencies. The dielectric permittivity at very low frequencies beyond this range (which is also needed in computations using the Lifshitz theory) is obtained by means of extrapolation of the measured optical data with the help of well tested Drude model

$$\varepsilon_D(\omega) = 1 - \frac{\omega_p^2}{\omega[\omega + i\gamma(T)]}. \quad (5)$$

Here  $\omega_p$  is the plasma frequency and  $\gamma(T) \ll \omega_p$  is the relaxation parameter. Equation (5) demonstrates that at very low, quasistatic, frequencies  $\omega \ll \gamma(T)$  the dielectric permittivity behaves as  $\varepsilon_D(\omega) \sim 1/\omega$ , as it must be in accordance with the Maxwell equations.

The first unexpected problem arising in the Lifshitz theory is that it violates the third law of thermodynamics (the Nernst heat theorem) when the interacting bodies are described by the dielectric permittivity (5) [41–43]. Specifically, it was shown [41–43] that the Casimir entropy

$$S_C(a, T) = -\frac{\partial \mathcal{F}_C(a, T)}{\partial T} \quad (6)$$

goes to a nonzero negative limit depending on the parameters of the system when the temperature vanishes. It was shown also [41–43] that the violation disappears when one neglects by the relaxation, i.e., suggests that  $\gamma(T) = 0$ . In this case the dielectric permittivity is described by the so-called plasma model

$$\varepsilon_p(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad (7)$$

which in fact valid only in the region of very high frequencies  $\omega \gg \gamma(T)$  characteristic for infrared optics. Keeping in mind that the fulfilment of the Nernst heat theorem is caused by the low-frequency behavior of  $\varepsilon$ , where not Eq. (7) but Eq. (5) is correct, the above facts should be considered as somewhat paradoxical. The discussion of this subject can be found, e.g., in Refs. [44, 45]. It was even suggested [46] that there might be profound difference in the reaction of a physical system to the real and fluctuating electromagnetic fields.

The second unexpected problem of the Lifshitz theory is that the theoretical Casimir force between metallic test bodies was found in drastic contradiction to the measurement data if the Drude model (5) is used at low frequencies. Alternatively, the predictions of the Lifshitz theory were found in excellent agreement with the measurement data when the dielectric permittivity of Au was extrapolated to low frequencies by means of the plasma model (7). This situation is illustrated in Fig. 2 where the predictions of the Lifshitz theory for the gradient of the Casimir force acting between Au-coated surfaces of a sphere and a plate are shown by the dark-gray and light-gray bands when the dielectric permittivities (5) and (7), respectively, were used in computations. The experimental data are shown as crosses whose arms indicate the total experimental errors. As can be seen in Fig. 2, the experimental data are in excellent agreement with the Lifshitz theory using the dielectric permittivity (7) (the so-called plasma model approach) and exclude the predictions of the Lifshitz theory using the dielectric permittivity (5) (the Drude model approach). Figure 2 is plotted by the results of the experiment [23] performed by means of a micromachined oscillator. Similar results leading to the same conclusions were later obtained by another experimental group by means of an atomic force microscope [47].

Thus, the Lifshitz theory combined with the plasma model was confirmed experimentally. It should be stressed that the constraints on the Yukawa-type corrections, presented in Sec. II, are obtained from the measure of agreement of the data with this theoretical approach. Therefore any additional arguments concerning its validity are highly desirable. First of all, we stress that the difference between the dark-gray and light-gray bands in Fig. 2 cannot be explained by the presence of some hypothetical Yukawa-type force acting between a sphere and a plate because of different dependences of these quantities on separation.

It was further hypothesized [48] that besides the Casimir force there might be some additional force between the sphere and the plate due to electrostatic patches caused by the grain structure of the polycrystal Au coatings, dust and contaminants on the surfaces.

The force gradient due to electrostatic patches is positive and leads to attraction. It was speculated [48] that when this force gradient is added to the theoretical prediction of the Drude model approach it might bring the resulting theoretical force in agreement with the experimental data. Then an apparent disagreement of the data with the Drude model approach would be explained.

In response to these arguments it was noted that the respective patches should be of rather large size which is in direct contradiction with the sizes of grains and the quality of surfaces used in the experiments [47]. However, in the end of 2012 final experimental confirmation of the plasma model approach was still missing providing possibility to cast doubts on the reliability of constraints obtained from the measure of agreement between experiment and theory. Situation has been changed in 2013 when the first measurements of the Casimir force between magnetic surfaces have been performed.

#### **IV. MEASUREMENTS OF THE CASIMIR FORCE BETWEEN FERROMAGNETIC AND CORRUGATED SURFACES LEAD TO NEW CONSTRAINTS ON NON-NEWTONIAN GRAVITY**

Although the Casimir interaction between ferromagnetic surfaces was predicted in 1971 [49], it was experimentally demonstrated for the first time quite recently [50, 51]. In Refs. [50, 51] the gradient of the Casimir force between two Ni-coated surfaces of a sphere and a plate was measured by means of an atomic force microscope. Measurement of the Casimir interaction between two magnetic surfaces is of fundamental importance because it sheds additional light on the validity of different approaches to the application of the Lifshitz theory and on the role of possible background effects, such as patch potentials, in theory-experiment comparison.

In Fig. 3 by the dark-gray and light-gray bands we show theoretical predictions for the gradient of the Casimir force between Ni-coated surfaces of a sphere and a plate calculated using the Drude and plasma model approaches, respectively. In the same figure, the experimental data with their total errors are shown as crosses. As can be seen in Fig. 3, the plasma model approach is again in excellent agreement with the data whereas the Drude model approach is experimentally excluded. In this respect Fig. 3 might be considered as similar to Fig. 2 related to the case of nonmagnetic (Au) surfaces. There is, however, the fundamental



difference between Figs. 3 and 2. The point is that for magnetic surfaces (Fig. 3) the Lifshitz theory combined with the Drude model predicts larger force gradients than the Lifshitz theory combined with the plasma model. This is exactly the opposite of that for nonmagnetic metals (Fig. 2) where the Drude model approach predicts smaller force gradients than the plasma model approach. Thus, if one suggests that there is some additional attraction due to patches (or some other background effect) between Au surfaces, which brings the Drude model approach in agreement with the measurement data, just this addition would bring the data for Ni surfaces in disagreement with both theoretical approaches.

One can conclude that measurements of the Casimir interaction between magnetic surfaces confirm the smallness of possible background effects in the aforementioned experiments using an atomic force microscope and micromachined oscillator, so that these effects do not influence on the comparison between experiment and theory. It is also confirmed that the Lifshitz theory using the plasma model at low frequencies correctly describes the Casimir interaction between metallic test bodies (in so doing the fundamental reasons behind this conclusion await for further investigation). The constraints on the Yukawa-type corrections to Newton's gravitational law, following from measurements of the Casimir force between magnetic surfaces, are obtained in Ref. [52]. They are in qualitative agreement with the constraints obtained in Refs. [23, 24] (see lines 2 and 3 in Fig. 1), but a bit weaker due to smaller density of Ni as compared to Au. However, the main importance of the experiment with Ni surfaces is that it has added confidence in all constraints on the Yukawa-type corrections to Newtonian gravity obtained from measurements of the Casimir interaction.

Another recent experiment is on measurement of the Casimir force between Au-coated sinusoidally corrugated surfaces of a sphere and a plate [53]. As opposite to Refs. [28, 29], here measurements were performed at various angles between corrugations. The corrugation periods on both test bodies were the same. The experimental results were found in good agreement with theoretical predictions using generalization of the Lifshitz theory for the case of nonplanar surfaces. The constraints on non-Newtonian gravity from this experiment were obtained in Ref. [52]. For this purpose the Yukawa-type force in the experimental configuration was calculated both exactly using Eq. (2) and approximately using the proximity force approximation with coinciding results [54, 55]. In Fig. 4 the solid line presents the most strong constraints on the Yukawa-type correction to Newtonian gravity which follow from the experiment of Ref. [53] at the angle between corrugations equal to  $2.4^\circ$ . In the same

figure the dashed lines 6 and 2 indicate the previously known strongest constraints in this interaction range obtained from measurements of the lateral Casimir force by means of an atomic force microscope and the gradient of the Casimir force by means of micromachined oscillator (in Fig. 1 these lines were shown as the solid lines 6 and 2, respectively). As is seen in Fig. 4, the new constraints are stronger than the previously known ones within the interaction region from  $\lambda = 11.6$  nm to  $\lambda = 29.2$  nm. The maximum strengthening by a factor 4 is achieved at  $\lambda = 17.2$  nm.

## V. CONCLUSIONS AND DISCUSSION

In this paper we have discussed new constraints on the Yukawa-type correction to Newton's gravitational law at short separations obtained recently from measurements of the Casimir interaction. These constraints were found from the measure of agreement between the experimental data and the fundamental theory of the van der Waals and Casimir forces developed by Lifshitz. It was shown that the comparison of the measurement data with this theory is a delicate problem. According to the experimental results, the relaxation properties of conduction electrons do not influence on the Casimir force and should not be included in theory-experiment comparison. Many persistent attempts to avoid this conclusion at the cost of some background effects or possible inaccuracy in calculations finally failed after recent demonstration of the Casimir interaction between ferromagnetic surfaces. At the moment the facts are known but the physical reasons behind them invite further investigation.

Measurements of the Casimir force continue to be very prospective for obtaining stronger constraints on the Yukawa-type corrections to Newtonian gravity at short separations. This was confirmed by the recent experiment with sinusoidally corrugated boundary surfaces performed at different angles between corrugations. The already achieved strengthening of the constraints by a factor of 4 from this experiment can be further improved due to some modifications in the measurement scheme. This shows that measurements of the Casimir force at a laboratory table continue to be an important source of information on the border between quantum physics and gravitation supplementary to information obtained from the

accelerator experiments, astrophysics and cosmology.

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- [1] A. A. Grib and S. G. Mamayev, Sov. J. Nucl. Phys. (USA) **10**, 722 (1970).
- [2] A. A. Grib and S. G. Mamayev, Sov. J. Nucl. Phys. (USA) **14**, 450 (1972).
- [3] A. A. Grib, B. A. Levitskii, and V. M. Mostepanenko, Theor. Math. Phys. (USA) **19**, 349 (1974).
- [4] A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, Gen. Relat. Grav. **7**, 535 (1976).
- [5] A. A. Grib, V. M. Mostepanenko, and V. M. Frolov, Theor. Math. Phys. (USA) **33**, 869 (1977).
- [6] A. A. Grib and Yu. V. Pavlov, Grav. Cosmol. **8**, 148 (2002).
- [7] A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, Fortschr. Phys. **28**, 173 (1980).
- [8] A. A. Grib, S. G. Mamayev, and V. M. Mostepanenko, *Vacuum Quantum Effects in Strong Fields* (Friedmann Lab. Publishing, St.Petersburg, 1994).
- [9] A. A. Grib, V. M. Mostepanenko, and V. M. Frolov, Theor. Math. Phys. (USA) **13**, 1207 (1972).
- [10] A. A. Grib, V. M. Mostepanenko, and V. M. Frolov, Theor. Math. Phys. (USA) **29**, 1131 (1976).
- [11] G. L. Klimchitskaya and V. M. Mostepanenko, Phys. Rev. D **87**, 125011 (2013).
- [12] E. Fischbach and C. L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer, New York, 1999).
- [13] S. Dimopoulos and G. F. Giudice, Phys. Lett. B **379**, 105 (1996).
- [14] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).
- [15] V. A. Kuzmin, I. I. Tkachev, and M. E. Shaposhnikov, Pis'ma v ZhETF **36**, 49 (1982) [JETP Lett. **36**, 59 (1982)].
- [16] V. M. Mostepanenko and I. Yu. Sokolov, Phys. Lett. A **125**, 405 (1987).
- [17] M. Bordag, G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, Oxford, 2009).
- [18] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Rev. Mod. Phys. **81**, 1827 (2009).
- [19] V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, and C. Romero, Phys. Rev. D **81**,

- 055003 (2010).
- [20] R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, *Eur. Phys. J. C* **51**, 963 (2007).
  - [21] E. Fischbach, D. E. Krause, V. M. Mostepanenko, and M. Novello, *Phys. Rev. D* **64**, 075010 (2001).
  - [22] B. W. Harris, F. Chen, and U. Mohideen, *Phys. Rev. A* **62**, 052109 (2000).
  - [23] R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, *Phys. Rev. D* **75**, 077101 (2007).
  - [24] R. S. Decca, D. López, E. Fischbach, D. E. Krause, and C. R. Jamell, *Phys. Rev. Lett.* **94**, 240401 (2005).
  - [25] M. Masuda and M. Sasaki, *Phys. Rev. Lett.* **102**, 171101 (2009).
  - [26] S. J. Smullin, A. A. Geraci, D. M. Weld, J. Chiaverini, S. Holmes, and A. Kapitulnik, *Phys. Rev. D* **72**, 122001 (2005).
  - [27] A. A. Geraci, S. J. Smullin, D. M. Weld, J. Chiaverini, and A. Kapitulnik, *Phys. Rev. D* **78**, 022002 (2008).
  - [28] H.-C. Chiu, G. L. Klimchitskaya, V. N. Marachevsky, V. M. Mostepanenko, and U. Mohideen, *Phys. Rev. B* **80**, 121402(R) (2009).
  - [29] H.-C. Chiu, G. L. Klimchitskaya, V. N. Marachevsky, V. M. Mostepanenko, and U. Mohideen, *Phys. Rev. B* **81**, 115417 (2010).
  - [30] S. G. Karshenboim, *Phys. Rev. D* **82**, 073003 (2010).
  - [31] S. K. Lamoreaux, *Phys. Rev. Lett.* **78**, 5 (1997).
  - [32] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, *Nature Phys.* **7**, 230 (2011).
  - [33] A. O. Sushkov, W. J. Kim, D. A. R. Dalvit, and S. K. Lamoreaux, *Phys. Rev. Lett.* **107**, 171101 (2011).
  - [34] D. Garcia-Sanchez, K. Y. Fong, H. Bhaskaran, S. Lamoreaux, and H. X. Tang, *Phys. Rev. Lett.* **109**, 027202 (2012).
  - [35] V. B. Bezerra, G. L. Klimchitskaya, U. Mohideen, V. M. Mostepanenko, and C. Romero, *Phys. Rev. B* **83**, 075417 (2011).
  - [36] G. L. Klimchitskaya and V. M. Mostepanenko, *Int. J. Mod. Phys. A* **26**, 3944 (2011).
  - [37] G. L. Klimchitskaya, M. Bordag, E. Fischbach, D. E. Krause, and V. M. Mostepanenko, *Int. J. Mod. Phys. A* **26**, 3918 (2011).

- [38] G. L. Klimchitskaya, M. Bordag, and V. M. Mostepanenko, *Int. J. Mod. Phys. A* **27**, 1260012 (2012).
- [39] M. Bordag, G. L. Klimchitskaya, and V. M. Mostepanenko, *Phys. Rev. Lett.* **109**, 199701 (2012).
- [40] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*. Pt. II (Pergamon Press, Oxford, 1980).
- [41] V. B. Bezerra, G. L. Klimchitskaya, and V. M. Mostepanenko, *Phys. Rev. A* **65**, 052113 (2002).
- [42] V. B. Bezerra, G. L. Klimchitskaya, and V. M. Mostepanenko, *Phys. Rev. A* **66**, 062112 (2002).
- [43] V. B. Bezerra, G. L. Klimchitskaya, V. M. Mostepanenko, and C. Romero, *Phys. Rev. A* **69**, 022119 (2004).
- [44] I. Brevik, J. B. Aarseth, J. S. Høye, and K. A. Milton, *Phys. Rev. E* **71**, 056101 (2005).
- [45] V. B. Bezerra, R. S. Decca, E. Fischbach, B. Geyer, G. L. Klimchitskaya, D. E. Krause, D. López, V. M. Mostepanenko, and C. Romero, *Phys. Rev. E* **73**, 028101 (2006).
- [46] V. M. Mostepanenko and G. L. Klimchitskaya, *Int. J. Mod. Phys. A* **25**, 2302 (2010).
- [47] C.-C. Chang, A. A. Banishev, R. Castillo-Garza, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, *Phys. Rev. B* **85**, 165443 (2012).
- [48] R. O. Behunin, F. Intravaia, D. A. R. Dalvit, P. A. Maia Neto, and S. Reynaud, *Phys. Rev. A* **85**, 012504 (2012).
- [49] P. Richmond and B. W. Ninham, *J. Phys. C: Solid St. Phys.* **4**, 1988 (1971).
- [50] A. A. Banishev, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, *Phys. Rev. Lett.* **110**, 137401 (2013).
- [51] A. A. Banishev, G. L. Klimchitskaya, V. M. Mostepanenko, and U. Mohideen, *Phys. Rev. B* **88**, 155410 (2013).
- [52] G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, *Phys. Rev. D* **87**, 125031 (2013).
- [53] A. A. Banishev, J. Wagner, T. Emig, R. Zandi, and U. Mohideen, *Phys. Rev. Lett.* **110**, 250403 (2013).
- [54] R. S. Decca, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, D. López, and V. M. Mostepanenko, *Phys. Rev. D* **79**, 124021 (2009).
- [55] E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, *Eur. Phys. J. C* **68**, 223 (2010).

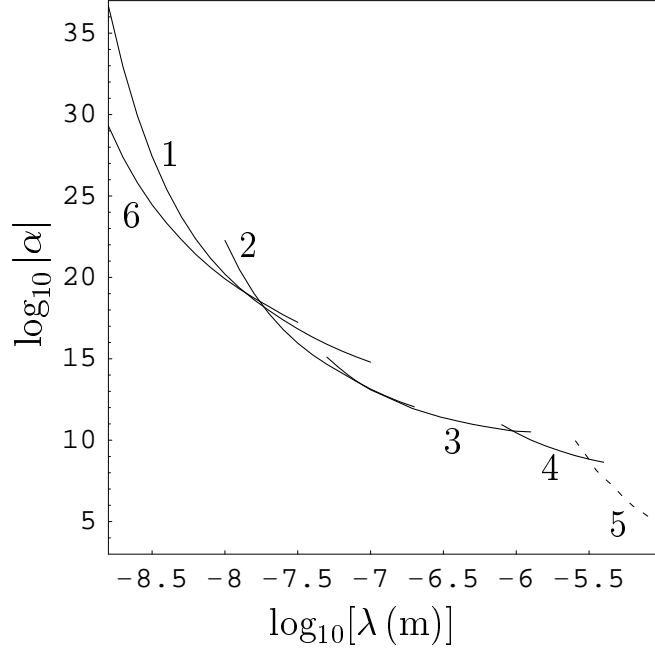


FIG. 1: Constraints on the Yukawa-type corrections to Newton's gravity from different experiments are shown by the lines 1–6 (see text for further discussion). The region of  $(\lambda, \alpha)$  plane above each line is prohibited and below each line is allowed.

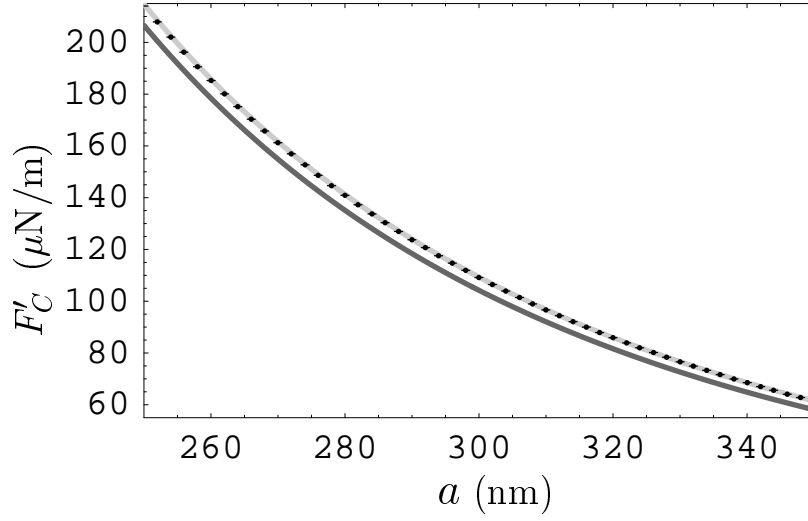


FIG. 2: The gradients of the Casimir force between Au surfaces measured by means of a micromachined oscillator versus separation are indicated as crosses. The dark- and light-gray bands show the theoretical predictions using the Drude and plasma model approaches, respectively.

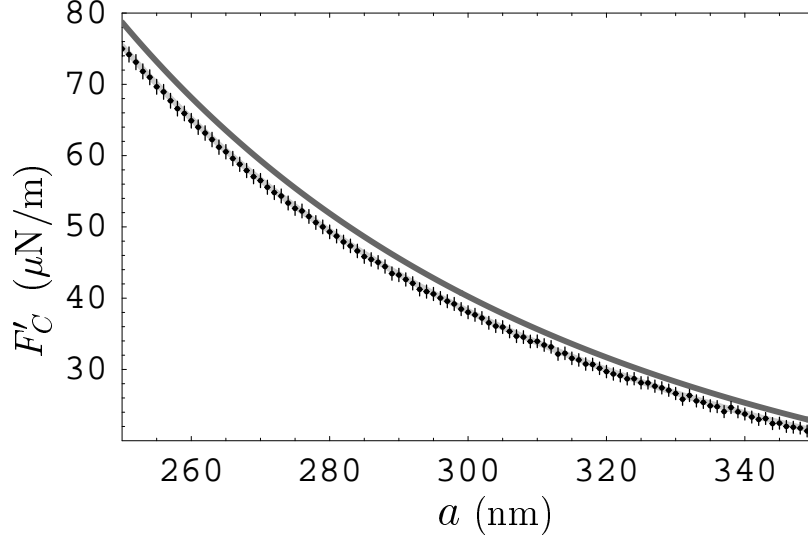


FIG. 3: The gradients of the Casimir force between Ni surfaces measured by means of an atomic force microscope versus separation are indicated as crosses. The dark- and light-gray bands show the theoretical predictions using the Drude and plasma model approaches, respectively.



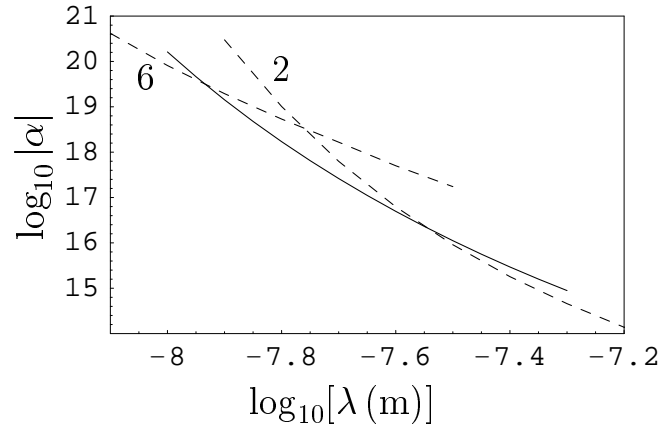


FIG. 4: Constraints on the Yukawa-type corrections to Newton's gravity from measurements of the Casimir force between sinusoidally corrugated surfaces as compared with other strongest constraints (see text for further discussion).